

Naked Singularities as Particle Accelerators II

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We generalize here our earlier results on particle acceleration by naked singularities. We showed recently [1] that the naked singularities that form due to gravitational collapse of massive stars provide a suitable environment where particles could get accelerated and collide at arbitrarily high center of mass energies. However, we focussed there only on the spherically symmetric gravitational collapse models, which were also assumed to be self-similar. In this paper, we broaden and generalize the result to all gravitational collapse models leading to the formation of a naked singularity as final state of collapse, evolving from a regular initial data, without making any prior restrictive assumptions about the spacetime symmetries such as above. We show that when the particles interact and collide near the Cauchy horizon, the energy of collision in the center of mass frame will be arbitrarily high, thus offering a window to the Planck scale physics. We also consider the issue of various possible physical mechanisms of generation of such very high energy particles from the vicinity of naked singularity. We then construct a model of gravitational collapse to a timelike naked singularity to demonstrate the working of these ideas, where the pressure is allowed to be negative but the energy conditions are respected. We show that a finite amount of mass-energy density has to be necessarily radiated away from the vicinity of the naked singularity as the collapse evolves. Therefore the nature of naked singularities, both at classical and quantum level could play an important role in the process of particle acceleration, explaining the occurrence of highly energetic outgoing particles in the vicinity of Cauchy horizon that participate in extreme high energy collisions.

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I. INTRODUCTION

Recently, an interesting observation was made by Banados, Silk and West [2], that black holes can accelerate infalling colliding particles to arbitrarily high energies in the center of mass frame around the horizon of an extremal Kerr black hole, provided certain restrictive conditions were imposed on the angular momenta of the particles. The plausibility and implications of this phenomenon in terms of realistic astrophysical black holes was also investigated [3]. Following the BSW paper, a number of works have now further investigated this effect in the context of an extremal charged spinning black hole, non-extremal rotating black holes, non-rotating charged black holes, stringy black holes, Kaluza-Klein black holes, examining similar possibilities in each case. It is also claimed that this may be a generic property of axially symmetric rotating black holes in a model independent way [4]. A general explanation to this effect of unbound acceleration has also been proposed [5].

In our recent work [1], we showed that the divergence of center of mass energy of colliding particles is a phenomenon not only associated with black holes, but also with naked singularities which are the final outcome of a continued gravitational collapse of a massive star. We considered the class of spherically symmetric, self-similar

gravitational collapse models, which lead to the formation of a naked singularity final state developing from a regular initial data. We considered collision between a highly energetic outgoing particle, emerging from a close vicinity of the naked singularity, and traveling close to the Cauchy horizon, with an ingoing particle. We showed that the center of mass energy of such a collision is arbitrarily large, depending on how close is the point of collision to the Cauchy horizon.

The limitation of our work, however, was that we considered there only the self-similar spherically symmetric spacetimes for the sake of simplicity and definitiveness. These models have served as a successful theoretical laboratory to test the possible outcomes for a complete gravitational collapse over past number of years, and to test the validity or otherwise of the cosmic censorship hypothesis. There have been several numerical as well as analytical investigations of self-similar gravitational collapse models, for different matter fields satisfying reasonable energy conditions, such as dust [6], ideal fluids with non-vanishing pressures [7], massless scalar fields [8, 9], and such others, leading to the formation of naked singularities in gravitational collapse from a regular initial data.

In fact, black hole and naked singularity formation in gravitational collapse models has been investigated in detail in recent years (see e.g. [10] and references therein for analytical results and [11] for numerical results), for a wide variety of physically realistic and relevant non-self-similar spherically symmetric situations as well. While most of the models investigated are spherically symmetric, certain non-spherical collapse scenarios have also been investigated (see e.g. [12]), where the collapse can

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result either in a black hole or naked singularity final state, depending on the nature of the initial data. The genericity and stability of such models, in the presence of non-spherical perturbations has also been considered in some detail. Thus a question that arises naturally is, whether or not the phenomenon of divergence of center of mass energy in a collision of particles is a generic phenomenon, or is it merely an artifact of various symmetries imposed on the spacetime such as the self-similarity or spherical symmetry. We address this issue in this paper. We show here the divergence of center of mass energy of colliding particles near the Cauchy horizon, without making any a priori assumptions about the spacetime symmetries such as those mentioned above.

This implies that the ultra-high-energy particle collisions is a phenomenon associated with naked singularities in a much more deeper manner, independent of various spacetime symmetries that are at times assumed for the sake of simplicity or definiteness. This phenomenon therefore deserves further investigation, and may have rather interesting physical consequences in terms of observability of the very high energy Planck scale physics.

We first make here some clarifying remarks about naked singularities before proceeding further. Whether or not naked singularities would occur in the real world we live in, is an unanswered question till this date. While these are hypothetical astrophysical objects, there is as yet no compelling observational evidence to confirm the existence of naked singularities unlike their black hole counterparts. However, considering the existence and recent emergence of very many gravitational collapse scenarios in general relativity, where the evolution of the collapsing matter cloud from a regular initial data leads to a naked singularity final fate for collapse, we may assume that the naked singularities could occur in various physical circumstances such as the final fate of a massive star, when it undergoes a complete gravitational collapse at the end of its life cycle on exhausting its internal nuclear fuel. It would be then of much physical interest to investigate the astrophysical consequences of their formation, also taking into account the possible quantum gravity effects these may cause [13], and the possible connection of the same with very highly energetic astrophysical phenomena, such as the gamma ray bursts and those related to the active galactic nuclei.

We show here that the naked singularities forming in gravitational collapse could provide us with a window into the new Planck scale physics, even far away from the actual singularity. This is because, the particles could go very close to a naked singularity and then emerge with very high velocities near the Cauchy horizon that the naked singularity created.

In the next Section II, we discuss the naked singularity and black hole formation in a general spacetime universe, and clarify the basic difference between these two fundamentally different final outcomes of a complete gravitational collapse of massive matter clouds in general relativity. Section III then investigates and shows the di-

vergence of center of mass energy in particle collisions near the Cauchy horizon, between the very high-energy particles coming from vicinity of the naked singularity, and the other ingoing particles. In Section IV, we examine and consider different physical possibilities and mechanisms as to how a naked singularity would possibly generate ultra-high energy particles, emerging from its vicinity. In Section V we then construct a toy collapse model to describe and demonstrate the ideas presented in Section IV. We show that when the repulsive nature of gravity in the vicinity of naked singularity is modeled by a large negative pressure, we naturally get highly energetic outgoing particles in the vicinity of Cauchy horizon that participate in high energy collisions. The final Section V gives a few concluding remarks and the possible future line of investigations.

II. GRAVITATIONAL COLLAPSE TO NAKED SINGULARITY

We now consider a general gravitational collapse scenario, where the collapse of a matter cloud proceeds from regular initial data, specified on an initial spacelike hypersurface. As a result of the complete gravitational collapse, a spacetime singularity is formed, and depending on the nature of the matter initial data and the velocities of the collapsing shells, the singularity could be either covered within a black hole, or it could be a naked singularity, where the event horizon does not hide the curvature divergence.

In the latter case above, where the evolution of the data in the complete gravitational collapse as described by Einstein equations results in the formation of naked singularity, the presence of the naked singularity is characterized by the existence of families of future directed outgoing null and timelike geodesics, which terminate in the past at the singularity, and in future they reach a faraway observer in the spacetime.

When the gravitational collapse ends in a naked singularity, this does not mean that the causality of the spacetime must be necessarily broken. The collapse may develop into a naked singularity but there need not be any closed timelike curves in the spacetime, and in that sense the spacetime is fully regular. This can be seen clearly in the dust collapse models, and many other gravitational collapse scenarios, where a naked singularity of collapse develops, and which have been analyzed in detail over past years [6].

In such a case, the spacetime necessarily admits a *Cauchy horizon*, which can be defined by the first null ray coming out of the singularity, in the case of spherical symmetry. In general, the future light cone of the very first point on the singularity curve is the Cauchy horizon in the spacetime (see Fig.1).

When the collapse terminates in a naked singularity, the spacetime is no longer *globally hyperbolic*, in the sense that there does not exist any global spacelike surface,

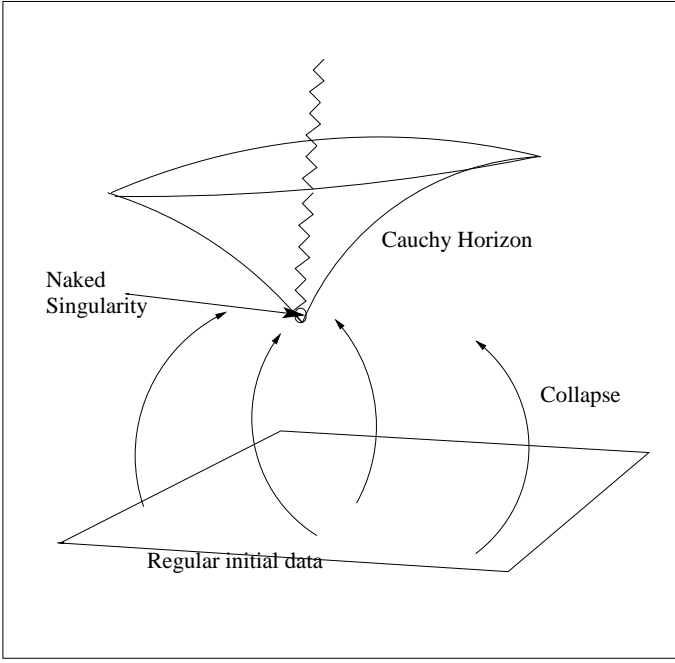


FIG. 1: The continual gravitational collapse starting from a regular initial data on a partial Cauchy hypersurface results in the formation of a naked singularity. The boundary of future light cone of the first point of naked singularity is the Cauchy horizon.

such that the initial data on the same would fully predict and determine the future as well as past evolutions in the spacetime for all times (for more details on global hyperbolicity and causal structure properties, see e.g. [14]).

In that case, it is the Cauchy horizon which marks the boundary of the region of the spacetime which is fully predictable from an initial surface. A naked singularity is therefore always characterized necessarily by the presence of a Cauchy horizon, which is in general a three dimensional null hypersurface in the spacetime, generated by null generators.

III. DIVERGENCE OF CENTER OF MASS ENERGY

We now consider the collisions between particles, the outgoing ones being those that emanate from close to the ultra-strong gravity region, that is the naked singularity, with those which are the infalling particles. The underlying scenario is that of a gravitational collapse that results in a naked singularity, and the collisions take place in the vicinity of the Cauchy horizon that is created by the naked singularity, and which is the first light cone coming out of the naked singularity. Our methodology is similar to that used in [5], to study the particle acceleration near an event horizon, which is also a three-dimensional null hypersurface in the spacetime.

Let l_α be a null vector field on the spacetime, which we take to be a generator of the Cauchy horizon of the spacetime that admits a naked singularity. Thus we have,

$$l_\alpha l^\alpha = 0. \quad (1)$$

We choose another null vector field n_α , thus satisfying

$$n_\alpha n^\alpha = 0. \quad (2)$$

We normalize n_α with respect to l_α so that

$$n_\alpha l^\alpha = -1. \quad (3)$$

We can then choose two mutually orthogonal spacelike vector fields b_i , $i = 1, 2$

$$b_{i\mu} b_j^\mu = \delta_{ij}, \quad (4)$$

which are orthogonal to the null vectors l_α, n_α . Thus,

$$l_\alpha b_i^\alpha = 0, n_\alpha b_i^\alpha = 0. \quad (5)$$

In terms of the four vector fields $l_\alpha, n_\alpha, b_{1\alpha}, b_{2\alpha}$, the spacetime metric $g_{\mu\nu}$ can then be written as,

$$g_{\mu\nu} = -l_\mu n_\nu - n_\mu l_\nu + \sigma_{\mu\nu}, \quad (6)$$

where

$$\sigma_{\alpha\beta} = b_{1\alpha} b_{1\beta} + b_{2\alpha} b_{2\beta}$$

is a spatial metric on the two-surface orthogonal to null vectors l_α, n_α (see [5] and [15]). Note that

$$\sigma_{\alpha\beta} l^\alpha = 0, \sigma_{\alpha\beta} n^\alpha = 0. \quad (7)$$

The velocity U^μ of a particle at any point in the spacetime can be written down now as a linear combination of these four vectors l, n, b_1, b_2 , since they form a basis for the tangent vector space,

$$U^\mu = A \left(\frac{l^\mu}{\alpha} + \beta n^\mu + \gamma^\mu \right), \quad (8)$$

where

$$\gamma^\mu = c_1 b_1^\mu + c_2 b_2^\mu$$

and A is a normalization factor chosen so that the velocity vector is normalized $U^\mu U_\mu = -1$. From (1),(2),(3),(6),(8), we can write,

$$A = \frac{1}{\sqrt{\frac{2\beta}{\alpha} - \sigma_{\alpha\beta} \gamma^\alpha \gamma^\beta}}. \quad (9)$$

We now consider two particles of masses m_1, m_2 , traveling with velocities $U_i, i = 1, 2$, which are given by,

$$U_i^\mu = A_i \left(\frac{l^\mu}{\alpha_i} + \beta_i n^\mu + \gamma_i^\mu \right), \quad (10)$$

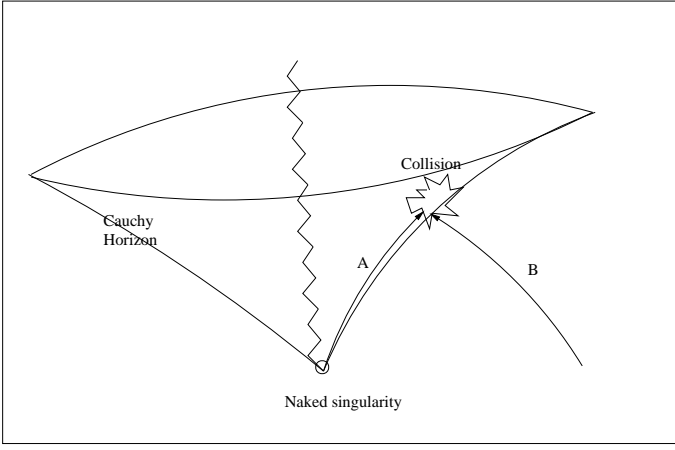


FIG. 2: A highly energetic particle "A" that has been accelerated by the naked singularity travels close to the Cauchy horizon. When it collides with an incoming particle "B", the center of mass energy is arbitrarily large, depending on how close is the point of collision to the Cauchy horizon.

with

$$A_i = \frac{1}{\sqrt{\frac{2\beta_i}{\alpha_i} - \sigma_{\alpha\beta}\gamma_i^\alpha\gamma_i^\beta}}.$$

When these particles interact at a given spacetime point, the energy of collision in the center of mass frame is given by (see [1] and [2]),

$$E_{cm}^2 = m_1^2 + m_2^2 - 2m_1m_2g_{\mu\nu}U_1^\mu U_2^\nu. \quad (11)$$

From (1),(2),(3),(6),(10),(11), we then finally obtain,

$$E_{cm}^2 = m_1^2 + m_2^2 + \frac{2m_1m_2\left(\frac{\beta_1}{\alpha_1} + \frac{\beta_2}{\alpha_2} - \sigma_{\alpha\beta}\gamma_1^\alpha\gamma_2^\beta\right)}{\sqrt{\frac{2\beta_1}{\alpha_1} - \sigma_{\alpha\beta}\gamma_1^\alpha\gamma_1^\beta}\sqrt{\frac{2\beta_2}{\alpha_2} - \sigma_{\alpha\beta}\gamma_2^\alpha\gamma_2^\beta}}. \quad (12)$$

We have considered here a situation where two particles collide near the Cauchy horizon (see Fig.2). One of the particles is an ingoing particle, and the other one is a highly energetic outgoing particle, which is coming from a close neighborhood of the singularity, and which therefore travels close to the Cauchy horizon. Such a particle can arise in a following way, as we described in our previous paper [1]. One can consider the region of spacetime before the formation of singularity, and the ingoing geodesics starting from a faraway region. After passing through the regular center these would emerge as outgoing geodesics. The outgoing particle close to the Cauchy horizon would be the ingoing geodesic, which just missed the naked singularity and emerged as an outgoing particle. Such ingoing particles in collapse may miss the singularity, if they had a small angular momentum or due to the small perturbations in geometry.

This particle (say particle 1) stays very close to the Cauchy horizon, and thus travels almost in the direction

of the horizon generator l_μ . For this to happen we must have

$$\alpha_1 \rightarrow 0. \quad (13)$$

Thus it can be clearly seen that the center of mass energy (12) diverges in the limit (13)

$$E_{cm}^2 \sim \frac{1}{\sqrt{\alpha_1}} \rightarrow \infty. \quad (14)$$

The value of α_1 signifies how close is an outgoing particle to the Cauchy horizon at the point of collision. Smaller the value of α , larger is the center of mass energy of collision between the ingoing and outgoing particles. Thus, by making α arbitrarily small, center of mass energy of collision can be made arbitrarily large.

At this point we would like to compare the results here with that of the previous work [1]. Earlier, we showed the divergence of center of mass energy between two identical massive particles following timelike geodesics, one of them ingoing and the other one outgoing. We assumed that the spacetime admitting naked singularity was spherical symmetric and also self-similar. These additional constraints imposed on the spacetime allowed us to integrate the geodesic equations and to explicitly write down the expression for the velocity of the particles. Then one can compute the center of mass energy, which was divergent in the limit where the point of collision approached the Cauchy horizon.

In the treatment here, we have considered the collision between two particles where their masses need not be the same. The condition that the particles follow geodesic motion is not imposed. This allows us to go beyond the test particle approximations, and consider a collision between particles that constitute the fluid which is a source of curvature of spacetime. As will be discussed later, the highly repulsive nature of the naked singularity, when quantum gravity effects are accounted for, could eject the initially collapsing fluid elements in radially outward direction with extraordinarily high energies. Particles constituting the fluid element would then travel close to what would have been the Cauchy horizon at classical level. These would then collide with incoming particles, which may be either the test or fluid particles. Since the fluid elements do not follow the geodesic motion in the presence of pressures, going beyond the assumption that the colliding particles follow geodesic motion would be useful. In fact, the only requirement necessary for the divergence of center of mass energy now is that, the velocity of one of the colliding particles, when expanded in the basis $(l^\mu, n^\mu, b_1^\mu, b_2^\mu)$, gets a dominant contribution from l^μ , which approaches the generator of the Cauchy horizon in the limit of the particle being closer and closer to the Cauchy horizon. This is again necessarily the requirement of the existence of particles with higher and higher energies, which would then travel closer and closer to the Cauchy horizon, coming from the vicinity of the naked singularity.

IV. PHYSICAL MECHANISMS OF GENERATION OF ENERGETIC PARTICLES

Collisions with arbitrarily high center of mass energies require the existence of extremely energetic particles, traveling close to the Cauchy horizon, which come from the vicinity of a naked singularity. In this section, we would like to discuss various mechanisms and physical scenarios in which such particles would be generated. The possible repulsive nature of gravity, as we discuss below, can play a crucial role in such acceleration mechanisms.

In general, gravity is always thought to be attractive. This fact is based on the purely Newtonian intuition rooted in our daily experience. However, gravity can exhibit a repulsive nature as well. Gravity turns out to be repulsive when the stress-energy tensor of the matter field, which is the source of spacetime curvature, violates the weak energy condition. The weak energy condition is violated when the pressures are sufficiently negative as compared to the energy density of the matter. In general relativity, since not only energy density but also the pressures gravitate, large negative pressures give rise to the repulsive gravity. The well-known examples of this phenomenon being the accelerated expansion of the universe due to a cosmological constant, and the inflation in early universe.

Similarly, the gravitational singularities that occur in general relativity, that are characterized by the divergence of Kretschmann scalar invariant $I = R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta}$, are commonly thought to be intrinsically and infinitely attractive in nature. However, the spacetime singularities can also exhibit a repulsive nature at times. We argue below that, as for the particle acceleration from the vicinity of singularities, an important role could be played here by the repulsive nature of naked singularities, both at the classical and quantum levels, in the process of acceleration of such particles. This, however, need not be the only mechanism to generate very high energy particles emanating from the vicinity of naked singularities.

A. Classical considerations

As we have already discussed earlier, infalling particles from infinity which just missed the naked singularity would emerge as highly energetic ultra-relativistic outgoing particles, and would travel close to Cauchy horizon. If the naked singularity that formed shows a repulsive behavior at classical level, particles then get further boosted up in energy, which we discuss in some detail in this section.

Gravity can show a repulsive character in the vicinity of naked singularities, although the weak energy conditions might still be respected. Such a repulsive nature, at the level of classical general relativity, is one of the intriguing and remarkable features associated with naked singularities. There are several ways to define and char-

acterize the repulsive nature of naked singularities. Many of these are specific to the coordinate system chosen. However, recently there has been a significant progress towards the coordinate invariant definition of repulsive nature of gravity (see [16], [17], [18]).

One can use the motion of test particles to study the repulsive nature of naked singularities. In stationary axially symmetric spacetimes, the radial motion can be described in terms of the effective potential, the explicit form of which depends on the coordinate system chosen. The effective potential has been demonstrated to be repulsive in the vicinity of naked singularities in the case of Reissner-Nordström, Kerr, and Kerr-Newmann naked singularities, which are inside the inner horizon of black-holes. This is also the case for naked singularities in these spacetimes, in the superextremal cases where the horizon is absent, thus exposing the singularity to the observer at infinity. It has also been suggested that the widening, instead of shrinking of the light-cones in approach to the naked singularity in the coordinates system that is adapted to the spacetime symmetries would be the indicator of repulsive nature of gravity [17]. This conjecture has been verified in the cases mentioned above as well. While the two approaches to characterize repulsive nature of gravity around naked singularities were observer dependent, an invariant way to define a repulsive gravity was proposed recently [18]. This involves the analysis of the eigenvalues of the curvature in the $SO(3,C)$ representation. The oscillation of the eigenvalues, while approaching the naked singularity along an arbitrary spatial direction marks the onset of the repulsive nature of gravity. This criterion has also been demonstrated to be consistent with cases discussed above in the context of other approaches. Thus gravity has been shown to exhibit repulsive nature in the vicinity of naked singularities in the stationary axially symmetric spacetimes.

While there has been an extensive analysis of repulsive nature of naked singularity in stationary spacetimes, there is not much attention paid to the singularities formed in the gravitational collapse. This is perhaps because so far the analysis has been dependent on the reference frames adapted to the spacetime symmetries, specifically the time translation symmetry, to analyze the geodesic motion of test particles. In the absence of a timelike killing vector in the case of gravitational collapse models, such an analysis is nontrivial and would be plagued by the gauge related issues. However, with the advent of gauge invariant techniques it might be possible to analyze the repulsive nature of gravity in non-stationary naked singularity models. We are currently investigating this issue, and would like to present our findings in future elsewhere.

At this point, however, we would like to conjecture that the naked singularities formed in the realistic gravitational collapse models would exhibit a repulsive nature in their vicinity, in the framework of classical general relativity. This would then play an important role in the particle acceleration mechanism in their vicinity. A

test particle infalling from infinity, marginally missing the naked singularity, and thus emerging as a highly energetic outgoing particle, would slow down very soon as it loses energy. Then it would deviate from its motion close to the Cauchy horizon, if gravity is not repulsive in the vicinity of the singularity. Thus, the desired collisions with arbitrarily large center of mass energies would occur in the small neighborhood around singularity. However, if gravity is repulsive then the outgoing particle would be further accelerated, and can thus continue to be highly energetic even in the region faraway from the singularity, as it travels closely near to the Cauchy horizon direction. Thus the collisions with arbitrarily high center of mass energies would happen in the region sufficiently faraway from the naked singularity, but close to the Cauchy horizon.

In a gravitational collapse starting from regular initial data resulting in the naked singularity, one might expect the transition where gravity would change its nature from being attractive to repulsive. In the beginning in the absence of singularity the gravity would be attractive. But on the onset of formation of naked singularity it would turn repulsive. It would be very interesting to study such a change in nature. It would play influential role in accelerating the test particle, when it is infalling, as well as when it travels outwards barely missing the singularity, by providing a further kick to provide a further acceleration to the particle so that it continues to travel faraway with high energy close to the Cauchy horizon.

B. Quantum gravity effects near naked singularity

Having discussed the role repulsive gravity around naked singularities might play in the mechanics of particle acceleration, we now consider the repulsive aspects associated with quantum gravity. Classical general relativity provides us with a good description of gravitational collapse up to the stage when densities and curvatures are small as compared to the Planck scale. But at Planck scales, the classical dynamics is no more a valid description of gravitational collapse, because the quantum mechanical effects must be taken into consideration as well. Thus we need a consistent quantum theory of gravity to deal with this situation. The spacetime is necessarily discrete or with a complicated topology at a deep down fundamental level. However, around the Planck scale in a certain regime, effective dynamics of the collapse can be at times described by modifying the Einstein equations, incorporating the non-perturbative corrections due to quantum gravity.

There have been a few attempts in this direction. We shall briefly discuss here one such work, describing the evolution of the collapsing object which would have classically led to a naked singularity, but taking into account the corrections due to quantum gravity resolves and removes the naked singularity [13]. In this work, the quantum theory of gravity that is used to modify classical

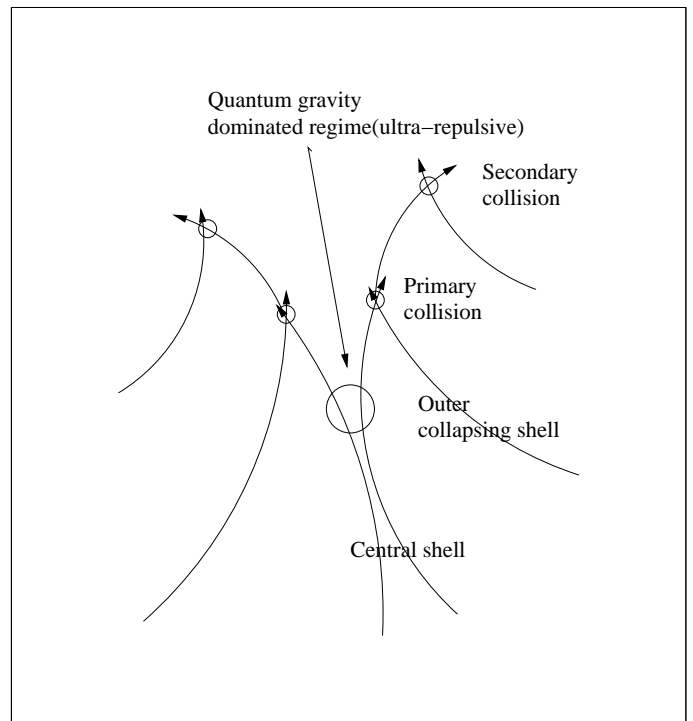


FIG. 3: A quantum gravitational collapse with resolution of the classical naked singularity. The central shell undergoes a collapse and rebound, having gone through the repulsive quantum gravity dominated regime. When highly energetic particles traveling outwards along the classical Cauchy horizon collide with the particles from outer collapsing shells, there are primary and then secondary collisions at arbitrarily high center of mass energies. A fireball is created, of finite extent, which is dominated by the Planck scale physics.

dynamics is the Loop quantum gravity formalism. This implements the non-perturbative quantization of gravity rewritten in Ashtekar variables in a consistent way. Loop quantum gravity has already successfully dealt with and resolved issues like singularity avoidance in cosmology [19], and such others. The model which gives rise to a naked singularity at classical level consists of a homogeneous scalar field with a suitable potential, with a finite spatial extent, which is matched to a generalized Vaidya exterior region at a constant comoving radius in a comoving reference frame. The potential is chosen in such a way that during the collapse the pressure is sufficiently negative so that the gravitational mass at any point in the spacetime is smaller than that of the physical radius in natural units. Therefore the trapped surfaces are not formed as the collapse evolves, and also there is an outgoing flux of energy in the Vaidya region. As the collapse proceeds, the singularity is formed in a finite proper time, as the density and curvatures blow up to infinity. The singularity is naked due to the avoidance of the trapped surfaces formation and there is an outward energy flux from the singularity which can be seen from infinity.

This classical scenario is modified by taking into ac-

count the non-perturbative semi-classical modification based on Loop quantum gravity. The dynamics at the semi-classical level which is valid in a certain regime around the Planck scale consists of modification of the matter Hamiltonian, thus modifying the spacetime density and pressures in the Einstein equations. The effective pressure is highly negative in comparison to the effective energy density and the effective equation of state for matter violates the weak energy conditions by an enormously large margin. The effective equation of state on the other extreme of validity of semi-classical regime turns out to be $\rho_{eff} = -9p_{eff}$, whereas the weak energy condition demands $\rho_{eff} + p_{eff} \geq 0$. This result is shown to be true for other matter fields as well [20]. There is a large outburst of energy, in the very final stages of the collapse, due to the extremely large negative pressure. As was mentioned earlier, the violation of energy conditions implies the repulsive nature of gravity. Since the pressure in this case is highly negative, gravity is ultra-repulsive. The density goes on decreasing, instead of blowing up like in the classical case, and the scale factor remains non-zero and well above the classical value till the breakdown of semiclassical regime. Beyond this regime the evolution would be governed by quantum difference equations.

Although the scenario described above is a toy model, describing the modified collapse dynamics of a homogeneous scalar field cloud with exterior Vaidya region, it gives an important insight so as to what the final collapse outcome might be in a generic collapse scenario following the complete quantum dynamics beyond the semiclassical regime. In general, it would be reasonable to expect that the naked singularity of collapse would be resolved due to quantum gravity effects. Due to ultra-repulsive nature of gravity in semi-classical and quantum regime, the fluid elements in the innermost core of the cloud that would have otherwise collapsed to the naked singularity for the first time, would be now thrown out and there would be large outburst of energy in the outward direction along what would have been classically the Cauchy horizon.

The particles constituting this ejected central fluid element would be highly energetic and travel at ultra-relativistic speeds along the Cauchy horizon. These, then would collide with the incoming particles, which could be either test particles or constituents of outer collapsing fluid elements, and the center of mass energy of such collisions would then be arbitrarily large (see Fig.3). The primary collisions would produce secondary relatively less energetic particles, which would be colliding with other particles with sufficiently high center of mass energies. Thus the collisions with divergent center of mass energies along the Cauchy horizon would generate a fireball of finite extent which would be dominated by quantum gravity effects and physics at ultra-high energies below the Planck scale, but still well above the center of mass energies realized in the terrestrial particle accelerators.

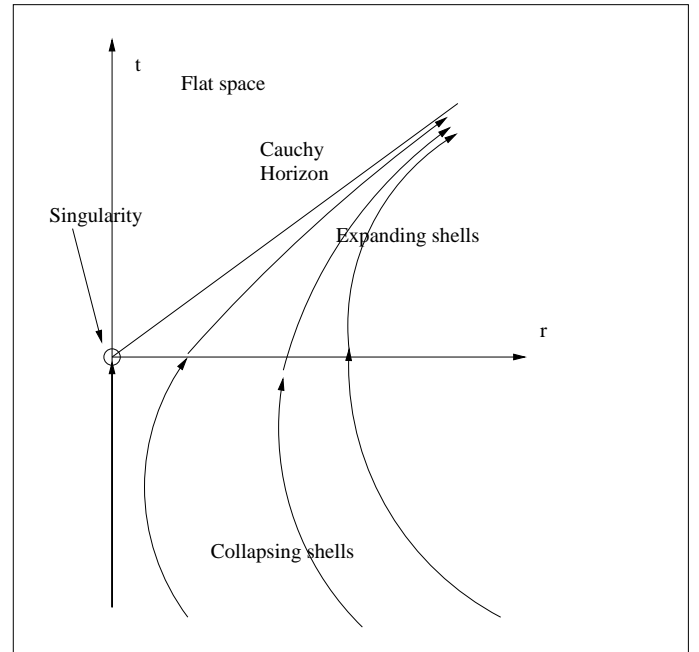


FIG. 4: Gravitational collapse of a fluid with large pressure inhomogeneities eventually turns into a dispersal with fluid elements moving outwards, eventually reaching closer and closer to the speed of light. The central shell, however, collapses to form a momentary singularity, which eventually disappears. All the non-central fluid elements asymptotically approach the Cauchy horizon of a naked singularity. When an incoming test particle interacts with a highly energetic fluid particle, the center of mass energy of collision is arbitrarily large. This is a schematic diagram describing this situation in the Schwarzschild-like coordinates.

C. Outward acceleration due to inhomogeneities

There is another interesting possibility where one might get highly energetic collisions along the Cauchy horizon (see Fig.4). Consider gravitational collapse of a fluid with non-vanishing pressure, for example, with a linear equation of state $p = k\rho$ with a sufficiently large k . Also consider a case of inhomogeneous collapse, where the density and pressure decrease as we move outwards from the center of collapse. There would then be a gradient of pressure pointing in the radially outward direction. Thus there is an outward force on the fluid element, thus accelerating it outwards. The density goes on increasing during the gravitational collapse, and if the distribution of matter is sufficiently inhomogeneous, then the pressure gradient also builds up. The fluid elements then experience larger and larger outward force. The central shell, however, undergoes a continual collapse since the regularity conditions demand the vanishing of pressure gradient at the center. Thus the center eventually collapses to singularity. However, if the trapped surfaces are not formed, then the non-central fluid shells eventually bounce back even before their density reaches the Planck scale since

the density and pressure gradients build up and attain large values and the large outward force causes the fluid elements to expand, following a contracting phase. All non-central shells accelerate outwards, reaching arbitrarily high velocities, and thus approach the Cauchy horizon of the central naked singularity. Solutions such as these are described, for example, in [7] within the context of self-similar models, and have been referred to as explosive solutions. In cosmology, such an effect has also been used to explain the observed cosmic acceleration of the universe, in the context of inhomogeneous scalar field models [21].

Infalling test particles from infinity which just miss the singularity are accelerated till they cross the momentary singularity in this case, and thereafter they would emerge as highly energetic outgoing particles in the Minkowski patch. They would then retain their energy, and thus continue to be ultra-relativistic in the region close to Cauchy horizon, till infinity.

When an incoming test particle interacts with the highly energetic fluid particle, or the outgoing test particle, in the vicinity of the Cauchy horizon, then in that case the center of mass energy of the collision is arbitrarily large, depending on how close is the point of interaction to the Cauchy horizon.

V. MASS-ENERGY EMISSION FROM THE VICINITY OF A TIMELIKE NAKED SINGULARITY

In the previous section we described various mechanisms of generation of highly energetic outgoing particles in the vicinity of a naked singularity that travel close to the Cauchy horizon and participate in the high energy collisions. We argued that the very nature of the naked singularity, at either the classical or semiclassical level, where large negative repulsive pressures could arise due to quantum gravitational corrections to the Einstein's equations, would generate highly energetic particles.

In this section we construct a gravitational collapse model to demonstrate these ideas. The collapse here describes the dynamical evolution of a matter cloud in the final stages of collapse leading to a timelike naked singularity. The cloud consists of a perfect fluid with a variable equation of state which is dependent both on time as well as spatial coordinates. Such a scenario appears to be reasonable in view of the fact that the collapse being a dynamical process leading to very high densities, the equation of state describing the relation between pressure and density in the collapsing cloud, which would be essentially dictated by a physical theory valid at that energy scale, is not to be expected to remain unchanged as collapse progresses.

Our model given below is purely classical and we assume that the pressure turns negative during the final stages of collapse, close to the formation of singularity.

Negative pressure can either be thought of as a model to describe an onset of a classical repulsive regime near the singularity, or it could also be interpreted as an indication of an occurrence of quantum gravity effects. We also assume that the spacetime is spherically symmetric since it would allow us to shed a light on the role played by a repulsive naked singularity in the particle acceleration process in a rather transparent way.

We shall see that under some circumstances the mass of the core of the collapsing star is radiated away by means of the negative pressures that originate near the center as the density approaches the singularity. Since the spacetime is spherically symmetric, no energy can be carried away in the form of gravitational waves. Therefore, if a finite amount of the infalling mass is ejected from the collapsing cloud, it must be in the form of photons or massive particles. In the final stages the cloud will therefore be matched to a generalized Vaidya exterior geometry, describing a space-time filled with outgoing radiation [22]. The outgoing radiation represents the energy carried out from the evaporation process of the central mass. At the time of formation of the singularity the entire mass of the cloud is radiated away, and therefore after the formation of the singularity the exterior spacetime will be described by the Minkowski metric. Particles escaping the vicinity of the singularity by this mechanism would have to be intrinsically highly energetic, since they originate in the region with extremely large curvature where quantum gravity is dominant and they would travel outwards in the vicinity of the Cauchy horizon (See Fig.5).

We will assume that

$$\frac{p(r, t)}{\rho(r, t)} = k(r, t) \quad (15)$$

where r and t are the comoving radius and time of the collapsing matter shells. In the following, primed quantities will denote derivatives with respect to r and dotted quantities will denote derivatives with respect to t .

From the Einstein equations we see that

$$p = -\frac{\dot{F}}{R^2 \dot{R}} \quad (16)$$

$$\rho = \frac{F'}{R^2 \dot{R}'} \quad (17)$$

where F is the Misner-Sharp mass of the system which is related to the amount of energy enclosed in the comoving radius r at the time t , and R is the physical radius of the shell that is shrinking to zero size. The collapse scenario is described by the condition that $\dot{R} < 0$.

The Misner-Sharp mass of the system is then defined by

$$F = R(1 - G + H) \quad (18)$$

where G and H are the metric functions.

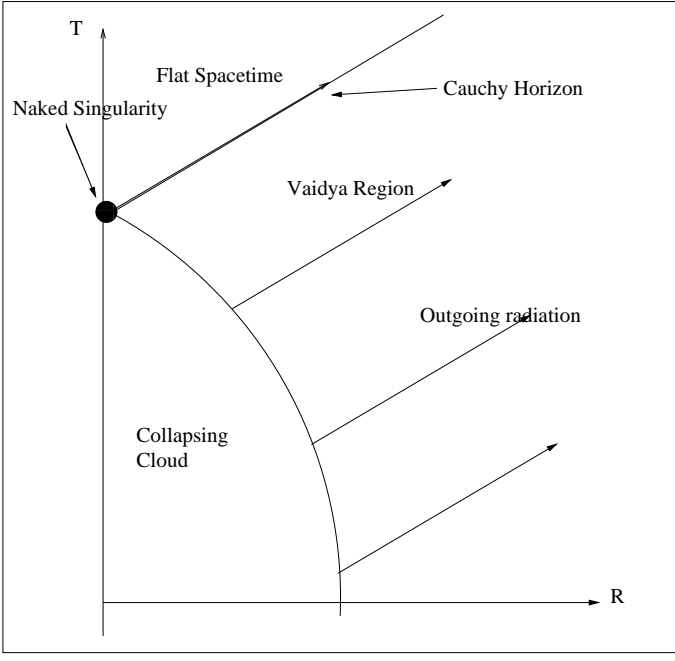


FIG. 5: A schematic diagram of gravitational collapse of a matter cloud to a naked singularity in Schwarzschild like coordinates (T, R) . The mass is radiated away due to the negative pressure in the late stages of collapse. The region outside the matter cloud is described by a generalized Vaidya metric. The cloud is radiated away completely leaving behind a flat spacetime. The same picture, given in comoving coordinates (t, r) , would depict the naked singularity as an extended timelike curve.

The spacetime metric in terms of functions $G(t, r)$, $H(t, r)$ and $R(t, r)$ can be written as

$$ds^2 = -\frac{\dot{R}^2}{H} dt^2 + \frac{R'^2}{G} dr^2 + R^2 (d\theta^2 + \sin^2\theta d\phi^2) \quad (19)$$

The remaining Einstein equations can then be written as

$$2\dot{R}' = R' \frac{\dot{G}}{G} + \dot{R} \frac{H'}{H} \quad (20)$$

$$\frac{p}{\rho + p} = -\frac{1}{2} \frac{R'}{\dot{R}} \frac{\dot{G}}{G} \quad (21)$$

We therefore have a system of six equations (the equation of state, the definition of the Misner-Sharp mass and the four Einstein equations), in seven unknowns (p, ρ, k, R, F, G, H) , therefore resulting in the freedom to choose one function [23]. Once k is chosen, we can obtain F , p and ρ as functions of R from equations (15), (16) and (17). Then equations (20) and (21) can be used to obtain G and H . The whole system then reduces to solving the equation of motion given by (18), which can be written as

$$\dot{R} = \pm \frac{k+1}{k} \frac{R'}{HG} \left(\frac{F}{R} + G - 1 \right) \quad (22)$$

Before we proceed we shall note two points. First of all the whole scenario has still a scaling degree of freedom, therefore by defining a function $v(r, t)$ from $R = rv$ and fixing the scale at the initial time to be $R(r, t_i) = r$, we can substitute the function R with v . Collapse will then be obtained for $\dot{v} < 0$, and the singularity is achieved for $v = 0$. At this point we note that approaching the singularity, v is monotonically decreasing in time, therefore allowing us to invert $v(r, t)$ and consider it as an alternative time coordinate. This represents the time at which the shell labeled by r reaches the event v . We shall therefore consider all functions as depending on r and v and rewrite the equations accordingly. The equation of motion, expressed in term of \dot{v} , can then be inverted as well to give $t(r, v)$.

Regularity of the matter model at the center of the cloud imposes some restrictions on the possible behaviors of the Misner-Sharp mass. It can be easily checked that we must have

$$F = r^3 M(r, v) \quad (23)$$

and that M may not contain linear terms in r in order to avoid cusps of the density at the center.

From the equation of state and equations (16) and (17), we derive the differential equation that must be satisfied by M as,

$$3kM + krM_{,r} + [v + (k+1)rw] M_{,v} = 0 \quad (24)$$

where $M_{,r}$ denotes derivatives of M with respect to r in the (r, v) coordinates, and $w(r, v)$ is given by v' expressed as a function of r and v ($w = v'(r, t(r, v))$). We shall note here that in principle w is not known because it requires the knowledge of v which comes from the integration of the equation of motion (22). Nevertheless in certain cases, corresponding to some specific choice of k , it might be possible to evaluate M .

We choose the function appearing in the equation of state, which is $k(r, v)$, as the free function of the system. Since all the quantities involved are well behaved (and at least \mathcal{C}^2) outside the singularity, we can always perform an expansion near the center and write

$$k(r, v) = k_0(v) + k_1(v)r + k_2(v)r^2 + \dots \quad (25)$$

This implies that p , ρ and M can be expanded accordingly. Restricting ourselves in a close neighborhood of the center (as it is relevant for our purposes) we can expand equation (24) and rewrite it, equating term by term. We thus obtain the following set of differential equations,

$$0 = 3k_0(v)M_0(v) + M_{0,v} \quad (26)$$

$$0 = 3k_1M_0 + 4k_0M_1 + M_{1,v}v + (1+k_0)w(0, v)M_{0,v} \quad (27)$$

$$0 = 3k_2M_0 + 4k_1M_1 + 5k_0M_2 + M_{2,v}v + k_1M_{0,v}w(0, v) + (1+k_0)M_{0,v}w_{,r}(0, v) + (1+k_0)M_{1,v}w(0, v) \quad (28)$$

where we have written $M = M_0(v) + M_1(v)r + M_2(v)r^2 + \dots$ and $w(r, v) = w(0, v) + w_{,r}(0, v)r + \dots$

The first equation can be solved for $M_0(v)$ once $k_0(v)$ is specified and gives,

$$M_0(v) = C_1 e^{-3 \int_v^1 \frac{k_0}{v} dv} \quad (29)$$

From regularity we see that $M_1 = 0$, and we can prove also that $w(r, v)$ must go like r near the center, thus imposing $w(0, v) = 0$. From the second equation we then get $k_1(v) = 0$.

In the third equation we use the freedom to choose $k_2(v)$ and take a specific class where

$$k_2 = (1 + k_0) k_0 \frac{w_{,r}(0, v)}{v} \quad (30)$$

This choice allows us to integrate the third equation to obtain

$$M_2(v) = \tilde{C}_2 e^{-5 \int_v^1 \frac{k_0}{v} dv} \quad (31)$$

We have therefore solved the system up to second order in r and $M(r, v)$ results as,

$$M(r, v) = C_1 e^{-3\Phi(v)} \left[1 + r^2 C_2 e^{-2\Phi(v)} \right] \quad (32)$$

where we have defined $C_2 = \frac{\tilde{C}_2}{C_1}$ and

$$\Phi(v) = \int_v^1 \frac{k_0(v)}{v} dv \quad (33)$$

Since we are interested in the behavior near the center, we have not considered here higher order terms. Nevertheless we shall notice that the freedom to choose the equation of state in the form of k allows us to potentially evaluate all other terms.

We see that positivity of C_1 is enough to ensure positivity of M near the center, while $C_2 < 0$ would imply that the density is decreasing radially outwards, thus imposing certain conditions on the boundary of the cloud once C_2 is set.

The set of Einstein equations is then solved once we integrate the equation of motion (22). Since in general it is not possible to integrate this equation explicitly, we shall restrict ourselves to a close neighborhood of the center $r = 0$ where we can write the inverse of equation (22) and integrate it to obtain $t(r, v)$ near the center as

$$t(r, v) = t(0, v) + \chi_1(v)r + \chi_2(v)r^2 + \dots \quad (34)$$

where the coefficients $\chi_i(v)$ are obtained from the expansion of t (which is always possible since the functions involved are generally at least \mathcal{C}^2). These quantities are related to the structure of the apparent horizon in the spacetime and the visibility of the singularity is determined by the same. The singularity curve is then given by $t_s(r) = t(r, 0)$ and describes the time at which the shell labeled by the comoving radius r becomes singular. In practice it can be shown that positivity of the

first non-vanishing term $\chi_i(0)$ is a necessary and sufficient condition for the outgoing geodesics to come out of the singularity [24].

In the present model, it is easy to check that regularity and the differential equations for M impose $\chi_1(0) = 0$. Therefore, the nature of the singularity is decided by the sign of $\chi_2(0)$, which is itself related to the energy density and pressures through M . Since M is given by (32) up to second order, and since we have the freedom to choose $k_i(v)$ for $i > 2$, it is not difficult to show that there are configurations that lead to the formation of a timelike naked singularity. Further, it can be shown that these configurations are related to the more physically realistic density profiles where ρ decreases as r increases (as obtained by choosing $C_2 < 0$). The singularity curve turns out to be increasing in time as r increases and the central shell is the first to become singular, as would be reasonable in a physically realistic scenario.

For the sake of clarity, let us briefly summarize the whole procedure followed above. Einstein's equations, together with the definition of the Misner-Sharp mass and the equation of state, constitute a set of 6 equations in 7 unknown (in (r, v) coordinates they are ρ, p, k, t, M, G, H), leaving us with the freedom to choose one function at will. Einstein's equations (16), (17), (20) and (21) can be integrated to give p, ρ, G and H . The system then reduces to solving two differential equations for $M(r, v)$ and $t(r, v)$, coming from equation (15) and (18), namely equation (24) and the inverse of equation (22). Restricting ourselves to a close neighborhood of the center we can expand all the quantities with respect to r . Equation (24) decouples from equation (22) for a specific choice of k , in which the coefficient $k_2(v)$ must be fixed. This choice allows us to integrate equation (24), thus providing the explicit form of $M(r, v)$ near the center. Using M as obtained in this manner, together with ρ, p, G and H as obtained from the other Einstein equations, we can integrate (22) for small values of r to obtain $t(r, v)$ as in equation (34). Therefore, integrating equation (22) in a close neighborhood of the center and writing $t(r, v)$ as in equation (34) solves completely the system of Einstein equations for small values of r , thus providing the space-time metric of the collapsing cloud approaching the singularity. The solution retains the freedom to choose the coefficient $k_0(v)$ from the expansion of the free function $k(r, v)$.

In order to evaluate the nature of the region surrounding the singularity curve we shall look for the behavior of the apparent horizon which determines the boundary of the trapped surfaces that can eventually form during collapse. The apparent horizon equation is given by

$$\frac{F}{R} = \frac{r^2 M(r, v)}{v} = 1 \quad (35)$$

which gives implicitly the curve of the apparent horizon as $r_{ah}(v)$.

Typically, in a pressure-free dust collapse, the mass is conserved and the apparent horizon must have radius

$r_{ah} = 0$ at the time of formation of the singularity (when $v = 0$). In that case, the singularity curve for all r with $r \neq 0$ is entirely trapped, and the central shell is the only one where the occurrence of the singularity can be eventually visible.

Nevertheless, in a perfect fluid collapse a wider variety of options is present. In the case when the mass is entirely radiated away during collapse as is the case above for the model we constructed here, we thus have $F(r, 0) = 0$ for $r > 0$, that is, for a finite range of comoving coordinate values of r . We thus see from equation (35) that no trapped surfaces form at all, all the way till the singularity formation for values $r > 0$, thus leaving a whole finite portion of the singularity curve to be visible and timelike. The resulting naked singularity is timelike with a range from $r = 0$ to a finite value of r . The key point is, when pressures are allowed to be negative, a finite portion of the non-central singularity at $r > 0$ can be visible as shown here.

As stated above, the condition in order to have the naked singularity to be timelike, is that F or equivalently M vanishes at $v = 0$. In this case we will have $\frac{F}{R} < 1$ in the limit of $v \rightarrow 0$ with $r \neq 0$. Then the region surrounding the singularity will not be trapped. This immediately implies that $M_{,v} > 0$ and therefore the pressure, which is given by $p = -\frac{M_{,v}}{v^2}$, as seen from equation (16), must be negative in this case. Since we have imposed an equation of state of the kind given by equation (15), negative pressures imply that in a neighborhood of the singularity we must have $k < 0$.

It is indeed possible, and plausible, that the gas cloud commences collapse with an equation of state describing classical positive pressures, and at a later stage, when the cloud approaches the singularity, the pressures become negative, probably triggered by the quantum effects. This finally leads to the evaporation of the core of the collapsing cloud and to the emission of highly energetic particles.

The free function k_0 describes at the lowest order in r the relation between the pressure and the density. Namely, it relates the central pressure and the central density at any given time, and its explicit behavior must be decided by the physical considerations and this will generally depend on the nature of the collapsing gas. Several scenarios where the mass F is radiated away during collapse can then be devised by choosing suitably the function $k(r, v)$, and we have seen that quantum effects could justify k to become negative at some stage before the formation of the singularity.

As an explicit example, we shall consider $k_0(v)$ to be written as in a series near the singularity

$$k_0(v) = k_{00} + k_{01}v + k_{02}v^2 + \dots \quad (36)$$

For simplicity we shall further assume that all higher order terms vanish. With the choice for k_0 made above

we obtain,

$$\frac{F}{R} = r^2 C_1 v^{-3k_{00}-1} e^{-3k_{01}v - \frac{3}{2}k_{02}v^2} \cdot \left[1 + r^2 C_2 v^{-2k_{00}} e^{-2k_{01}v - \frac{2}{2}k_{02}v^2} \right] \quad (37)$$

The requirement that $\frac{F}{R} < 1$ near the singularity imposes the following restriction on k_{00} ,

$$k_{00} \leq -\frac{1}{3} \quad (38)$$

which is a condition for avoidance of trapped surfaces, and which gives rise to the formation of a non-central naked singularity. We can choose k_{01} and k_{02} so that the pressure is positive to begin with, at the onset of collapse for large values of v and at the initial time. The pressure then decreases during collapse and turns negative at the later stages of collapse, eventually leading to the formation of a naked singularity.

We can see that in the limiting case where $k_{00} = -\frac{1}{3}$, then in the limit of approach of singularity we get,

$$\lim_{v \rightarrow 0} \frac{F}{R} = r^2 C_1 \quad (39)$$

which implies a finite radius for the trapped surfaces at all times. This in turn means that we must take the boundary of the collapsing cloud according to

$$r_b < \frac{1}{\sqrt{C_1}} \quad (40)$$

in order to avoid trapped surfaces during the whole collapse. In the case where $k_{00} < -\frac{1}{3}$, the apparent horizon simply does not form as the singularity is approached, thus suggesting that the boundary chosen in order to avoid trapped surfaces can be arbitrarily large in this case.

As we have said, the presence of negative pressure requires that the function M vanishes at the singularity, which implies that the whole mass of the core must be radiated away during the last stages of collapse. This shows how in these collapse models a finite amount of energy escapes away from the vicinity of the naked singularity. In the classical picture, this mechanism requires the presence of negative pressures, which can be justified by quantum effects occurring near the singularity, as it has already been shown both in the semiclassical approximation as well as in some simple toy models in Loop Quantum Gravity [20]. Particles escaping the vicinity of the singularity by this mechanism would have to be intrinsically highly energetic since they originate in the region where quantum gravity is dominant. These particles, when traveling close to the Cauchy horizon, would retain their energy even at large distances from the singularity as they travel in the region of spacetime which can be described by a metric which is Minkowski with small perturbations.

In this sense, such a timelike naked singularity can provide a window on the physics of Planck scale. These particles would travel in a region close to the Cauchy horizon and participate in the large center of mass energy collisions with ingoing particles, as was described in section II.

VI. CONCLUSION

We showed here that naked singularities that form in gravitational collapse of massive stars, evolving from a regular initial data, can act as cosmic super-colliders. Particles interact and collide near the Cauchy horizon of the naked singularities with arbitrarily high center of mass energies. This might provide us the natural laboratories to study the physics of collisions with center of mass energies significantly large, as compared to those realized in terrestrial particle accelerators, and these could go all the way high, up to the Planck scale in an astrophysical setting. This scenario is subject to the existence of naked singularities in nature, as produced in astrophysical events such as collapse of massive stars.

Given the existence of many solutions to Einstein equations admitting naked singularities in general relativistic gravitational collapse of physically realistic matter fields, it might be reasonable to expect that naked singularities might occur in nature, and if they do so, then they would turn out to be a boon for our understanding of various physical processes occurring at ultra-high energies, not realizable in any other laboratory or astrophysical settings, apart from perhaps the very early universe. Naked singularities would accelerate particles to arbitrarily high energies, which travel close to the Cauchy horizon, and these particles would participate in high energy collisions. We also considered and discussed here various physical mechanisms responsible for production of such particles by naked singularities. Incoming particles falling inwards from infinity just missing the singularity, and then following the outward trajectory, would be extremely energetic. They perhaps might get an additional boost due to the repulsive nature of naked singularities at classical and quantum level. They would travel close to the Cauchy horizon and collide with incoming particles at large center of mass energies.

During the final stages of gravitational collapse, the density and curvatures reach the Planck scale, where dynamics of collapse is ruled by quantum gravity. Quantum gravity is known to be ultra-repulsive and is expected to resolve the classical naked singularity. Calculations carried out in semiclassical regime strongly indicate that there would be a strong outburst of extremely large

amount of energy and central fluid elements might be thrown apart at gigantic energies in full quantum theory of gravity, when the discrete evolution is taking into account. When the ultra-relativistic fluid particles bounce back and travel close to what would have been a Cauchy horizon in classical picture, they collide with fluid particles from the outer shells undergoing collapse. Thus there would be large number of collisions with arbitrarily large energies, followed by more collisions between energetic particles emitted from the primary collisions and the outer fluid elements in the collapsing cloud. This can generate a quantum fireball, physical processes occurring in which would be governed by quantum gravity.

We also proposed another situation where collapse of the central shell would yield momentary singularity but the noncentral shell, because of large inhomogeneity in pressure, would be blown apart to ever increasing velocity, eventually reaching asymptotically to the Cauchy horizon of the singularity, where collision with ingoing particles would reach large center of mass energies. Thus, we demonstrated that the naked singularity, if it were to occur in nature, could turn out to be a boon for understanding phenomenon at and around the Planck scale.

We have described here a model to demonstrate how the repulsive nature of gravity, as modeled by large negative pressure in the vicinity of naked singularity, accounts for an outward flux of energy in the form of highly energetic photons or massive particles. We studied the final stages of a collapsing cloud consisting of an ideal fluid with a variable equation of state. We chose an equation of state so that the pressure becomes negative in the vicinity of the naked singularity. This phenomenon is well motivated from the quantum gravity effects near the singularity. The existence of negative pressure essentially implies the decrease in the Misner-Sharp mass of the cloud, in the form of an outward flux of energy which can be modeled by exterior generalized Vaidya spacetime. If the mass is radiated sufficiently fast then the formation of trapped surfaces can be avoided, thus exposing the singularity. The entire mass of the cloud can also be radiated away, leaving behind a flat spacetime. There is an outgoing radiation from close to naked singularity which consists of highly energetic photons or massive particles, since it originates in the region with extremely high curvatures dominated by quantum gravity. When these particles collide and interact with ingoing particles, the center of mass energy of collision would be extremely large, offering a window to new physics at energies inaccessible at terrestrial particle accelerators.

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